Dilaton Quantum Gravity
A Functional Renormalization Group Approach

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### Outline

1. **Introduction: Asymptotic Safety & Quantum Einstein Gravity**
   - Nonrenormalizibility and Asymptotic Safety
   - Beyond Perturbation Theory: Functional Renormalization
   - Challenges & Open Questions

2. **Scalar Tensor Theories and Dilatation Symmetry**
   - Dynamical Constants
   - Results

3. **Summary and Outlook**
Renormalizibility and Quantum Gravity

- Einstein-Hilbert action

\[ \Gamma [g_{\mu\nu}] = \frac{1}{16\pi G_N} \int d^d x \sqrt{g} \left( 2\Lambda - R[g_{\mu\nu}] \right) \]

- mass dimensions of the couplings

\[ \text{dim}(\Lambda) = 2 \]
\[ \text{dim}(G_N) = 2 - d \]

\[ \implies \text{Perturbatively not renormalizable if } d > 2. \]
Alternative Approaches

Introduction of radically new concepts
(Super) String Theory, Loop Quantum Gravity, ...

What if we give up perturbation theory instead?
The Asymptotic Safety Scenario
Alternative Approaches

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What if we give up perturbation theory instead?
The Asymptotic Safety Scenario
Asymptotic Safety

Weinberg, 1979

A theory is said to be asymptotically safe if the essential coupling parameters approach a fixed point as the momentum scale of their renormalization point goes to infinity.

Working translation

Quantum gravity is considered asymptotically safe if the UV-critical surface is finite dimensional and if the dimensionless coupling constants cease to increase if the momentum scale $k$ goes to infinity, but approach an ultraviolet fixed point instead.
Wetterich Equation

$$\partial_t \Gamma_k = \frac{1}{2} \text{STr} \left[ \frac{1}{\Gamma_k^{(2)} + \mathcal{R}_k} \partial_t \mathcal{R}_k \right] , \quad t = \log(k/k_0)$$
Wetterich Equation

\[ \partial_t \Gamma_k = \frac{1}{2} \text{STr} \left[ \frac{1}{\Gamma_k^{(2)} + \mathcal{R}_k} \partial_t \mathcal{R}_k \right], \quad t = \log(k/k_0) \]
The Flow Diagram of Quantum Einstein Gravity

Reuter, 1998

Coupling Constants approach a nontrivial UV Fixed Point

Prospect of Gravity being Asymptotically Safe
The Flow Diagram of Quantum Einstein Gravity

Christiansen, Litim, Pawlowski, Rodigast, 2012

Stable Infrared Scenarios

→ Prospect of UV and IR consistent theory
# Asymptotically Safe Quantum Gravity

<table>
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<tr>
<th>FRG Technicalities</th>
<th>Coupling to SM</th>
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<td>- Regulator Dependence</td>
<td>- Yang Mills Theory</td>
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<td>- Background Dependence</td>
<td>- Background Dependence</td>
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<td>- Truncation Stability</td>
<td>- Asymptotic Freedom</td>
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- **Infrared Limit**
  - Trajectory UV $\rightarrow$ IR
  - GR as limiting case?

- **Hierarchy Problem**
  - $M_{SM} \approx 10^2$ GeV
  - $M_{Planck} \approx 10^{19}$ GeV
Dynamical Constants I: Scalar-Tensor Theories

\[ \Gamma_k [g_{\mu \nu}] = \frac{1}{16\pi G_{N,k}} \int d^d x \sqrt{g} (2\Lambda_k - R[g_{\mu \nu}]) \]

\[ \Gamma_k [g_{\mu \nu}, \chi] = \int d^d x \sqrt{g} \left( V_k[\chi] - F_k[\chi] R[g_{\mu \nu}] + \frac{1}{2} g_{\mu \nu} \partial^\mu \chi \partial^\nu \chi \right) \]

Assumptions:

- \( O(R) \) truncation
- no wave function renormalization
- curved background spacetime & optimized cut-offs
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Dynamical Constants II: Dilatation Symmetry

- Dilatations ↔ Conformal Transformations
  \[ g_{\mu\nu}(x) \rightarrow \Omega(x)g_{\mu\nu}(x) \]
  with \( \Omega = \text{const} \).
- Dilatation ↔ global resetting of the physical scale
- Dilatation Symmetry ↔ Physical Scale is introduced only by expectation value of the scalar field
- Arising Goldstone Boson: Dilaton

Dilatation Symmetric Actions

\[ \Gamma \text{ is invariant under dilatations} \]
\[ \Updownarrow \]
all couplings have scaling dimension 0.
\[ (d = 4: F \propto \chi^2, V \propto \chi^4) \]
Limiting Cases

dimensionless field $\tilde{\chi}$ ($d = 4 : \tilde{\chi} = k^{-2}\chi$)

**Ultraviolet: $\tilde{\chi} \to 0$**
- $\chi \to 0$
- $k \to \infty$
- $V, F$ power series in $\tilde{\chi}^2$
- Percacci, 2009

**Infrared: $\tilde{\chi} \to \infty$**
- $\chi \to \infty$
- $k \to 0$
- $V, F$ power series in $\tilde{\chi}^{-2}$
- current project, 2012
The Infrared Limit I: Expansions

- Dilatation Symmetric Couplings have vanishing $\beta$-functions
- Closed set of flow equations to each order in $\tilde{\chi}^{-2}$
- Fixed point value only in constant order nontrivial

$\Rightarrow$ stable Einstein-Hilbert infrared limit
The Infrared Limit II: Dilatation Symmetry

**Dilatation Symmetric Infrared Scenario**

\[ V = 0 \text{ and } F = \xi \chi^2 \]

\[ \Gamma_{k \rightarrow 0} = \int d^d x \sqrt{g} \left( \frac{1}{2} g_{\mu \nu} \partial^\mu \chi \partial^\nu \chi - \xi \chi^2 R \right) \]

- Flow diverges at conformal coupling parameters
- Weyl Anomaly is realized
Summary and Outlook

- Multifaceted evidence for *asymptotic safety* scenario
- Dilatation symmetric *infrared limit*

- Stability surveys
- Continuation past asymptotic cases

Thank you very much!